

Correction

S. Joucla and B. Yvert. 2009. The “mirror” estimate: an intuitive predictor of membrane polarization during extracellular stimulation. *Biophys. J.* 96:3495–3508.

THEORETICAL BACKGROUND

In the theoretical section, Eq. 11, initially written as

$$V_m(s) = -V_{\text{ext}}(s) + \frac{\int_{s=-\frac{l}{2}}^{\frac{l}{2}} V_{\text{ext}}(s)/\lambda(s)^2 ds}{\int_{s=-\frac{l}{2}}^{\frac{l}{2}} 1/\lambda(s)^2 ds}$$

should be replaced with

$$V_m(s) = -V_{\text{ext}}(s) + \frac{\int_{s=-\frac{l}{2}}^{\frac{l}{2}} g_l(s) \times d(s) \times V_{\text{ext}}(s) ds}{\int_{s=-\frac{l}{2}}^{\frac{l}{2}} g_l(s) \times d(s) ds}.$$

The proof is given in the following Appendix, which replaces the initial version given in the published article.

APPENDIX

In this appendix, we derive the mirror approximation in the case of a nonuniform fiber (Eq. 11). We consider the case where the fiber diameter and the leakage conductance are different for each compartment k in $0 \dots N$. Equation 1 can then be rewritten as

$$\begin{aligned} C_m^{(k)} \times \frac{dV_m^{(k)}}{dt} + G_1^{(k)} \times V_m^{(k)} &= \frac{V_{\text{int}}^{(k-1)} - V_{\text{int}}^{(k)}}{R^{(k,k-1)}} + \frac{V_{\text{int}}^{(k+1)} - V_{\text{int}}^{(k)}}{R^{(k,k+1)}} \quad k = 1 \dots N-1 \quad (\text{a}) \\ C_m^{(0)} \times \frac{dV_m^{(0)}}{dt} + G_1^{(0)} \times V_m^{(0)} &= \frac{V_{\text{int}}^{(1)} - V_{\text{int}}^{(0)}}{R^{(0,1)}} \quad k = 0 \quad (\text{b}) \cdot \\ C_m^{(N)} \times \frac{dV_m^{(N)}}{dt} + G_1^{(N)} \times V_m^{(N)} &= \frac{V_{\text{int}}^{(N-1)} - V_{\text{int}}^{(N)}}{R^{(N,N-1)}} \quad k = N \quad (\text{c}) \end{aligned} \quad (\text{A1})$$

At the steady state, all capacitive terms drop, and, summing all these equations, we get

$$\begin{aligned} \sum_{k=0}^N G_1^{(k)} \times V_m^{(k)} &= \sum_{k=0}^{N-1} \frac{V_{\text{int}}^{(k+1)} - V_{\text{int}}^{(k)}}{R^{(k,k+1)}} + \sum_{k=1}^N \frac{V_{\text{int}}^{(k-1)} - V_{\text{int}}^{(k)}}{R^{(k,k-1)}} \\ &= \sum_{k=0}^{N-1} \frac{V_{\text{int}}^{(k+1)} - V_{\text{int}}^{(k)}}{R^{(k,k+1)}} + \sum_{k=0}^{N-1} \frac{V_{\text{int}}^{(k)} - V_{\text{int}}^{(k+1)}}{R^{(k,k+1)}} \\ &= 0 \end{aligned} \quad (\text{A2})$$

We now consider the case where the mirror estimate applies, i.e., for which the space constant $\lambda(k) = [d(k)/(4 \times \rho_i(k) \times g_l(k))]^{1/2}$ is high in every compartment. In such case, we can write $V_m(k) = -V_{\text{ext}}(k) + c_2$ (Eq. 6). Introducing this expression into Eq. A2, we get

$$c_2 = \frac{\sum_{k=0}^N G_1^{(k)} \times V_{\text{ext}}}{\sum_{k=0}^N G_1^{(k)}} = \langle V_{\text{ext}} \rangle, \quad (\text{A3})$$

with $G_1(k) = g_l(k) \times \pi \times d(k) \times \Delta s$.

In conclusion, in the (general) nonuniform case, assuming that all $\lambda(k)$ are “large enough”, the membrane potential is the mirror of the extracellular potential centered on its spatial mean value weighted by the squared inverse of the space constants:

$$V_{\text{m}}^{(k)} = -V_{\text{ext}}^{(k)} + \frac{\sum_{j=0}^N g_1^{(j)} \times d^{(j)} \times V_{\text{ext}}^{(j)}}{\sum_{j=0}^N g_1^{(j)} \times d^{(j)}}, \quad (\text{A4})$$

which becomes, in the continuous case

$$V_{\text{m}}(s) = -V_{\text{ext}}(s) + \frac{\int_{s=-\frac{1}{2}}^{\frac{1}{2}} g_1(s) \times d(s) \times V_{\text{ext}}(s) ds}{\int_{s=-\frac{1}{2}}^{\frac{1}{2}} g_1(s) \times d(s) ds}. \quad (\text{A5})$$